REDISTRIBUTION OF ARBITRARY SYSTEM OF INTERNAL STRESSES IN A NORMALLY INCIDENT SHOCK WAVE

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Laid-on or fused explosive charges are currently widely used to remove residual stresses in welded plates. The practical and physical aspects of this phenomenon are discussed in a number of papers [1-3].

In particular, in [1], the symmetrical case of initial stresses, $\sigma_{y0} = \sigma_{z0}$ and $\sigma_{x0} = 0$, was examined while studying the trajectory of the point representing the stress–strain state of the substance in the space of the principal stresses during the process of explosive loading and unloading. (The shock wave front is parallel to the surface of the metal, the axes σ_y and σ_z lie in the plane of the surface, and the axis σ_x is perpendicular to the surface. The metal is assumed to be elastoplastic.) In this case, for any initial stresses, within the limits of elasticity, the straight line of the elastic stress reaches the yield surface at points belonging to some definite straight line, whose projection on the σ_x, σ_y surface is the straight line QQ shown in Fig. 1 (in the case of a stretching wave, this is the straight line PP). The quantities OD = OE = OF = OG = σ_s, the straight line OS is the projection of the hydrostatic axis, and σ_s is the dynamic yield stress. Under further loading, the image point moves along the straight line QQ.

For $\sigma_{y_0} \neq \sigma_{z_0}$, the loading trajectories are more complicated; as will be shown below, these trajectories in the plastic region form a family of curves having as an asymptote for large σ_x straight lines whose projections on the plane σ_x, σ_y are PP and QQ.

The purpose of this paper is to determine these trajectories and based on them, the limits of applicability of the scheme presented in Fig. 1 in the case of unsymmetrical initial stresses. The possibility of using this scheme is determined by the initial stresses σ_{y_0} and σ_{z_0} and the magnitude of the load σ_{xk}.

Using the associated law for the flow [4]

$$d\varepsilon_{xp}/S_x = d\varepsilon_{yp}/S_y = d\varepsilon_{zp}/S_z = |d\varepsilon_p|/|\mathbf{S}|, \qquad (1)$$

it is possible to obtain the system of equations

$$\frac{4}{3}Gd\varepsilon_x = \frac{dS_x}{1-\left(\frac{3S_x}{2\sigma_s}\right)^2}, \quad -\frac{2}{3}Gd\varepsilon_x = \frac{dS_y}{1+\frac{9S_xS_y}{2\sigma_s^2}} = \frac{dS_z}{1+\frac{9S_xS_z}{2\sigma_s^2}}, \qquad (2)$$

which permits finding the dependences σ_x, σ_y, and σ_z on the load parameter ε_x. The possible load trajectories in the elastic and plastic regions, corresponding to solutions of the system (2), are shown in Fig. 2 in the combined coordinates of the planes σ_x, σ_y and σ_x, σ_z. Here, the region bounded by the straight lines RR and R_1R_1 is the projection of the flow cylinder, while the straight line QQ is the asymptote of the trajectories for $\sigma_x \to -\infty$; σ_{x_1} corresponds to the onset of plastic flow. It is more convenient to examine the process in the coordinates σ_x, σ_m, where $\sigma_m = (1/2)(\sigma_y + \sigma_z)$. The corresponding trajectories are shown in Fig. 3.

The loading is described as follows.

1. The starting point belongs to the segment AD, $\sigma_{m0} = (1/2)(\sigma_{y0} + \sigma_{z0})$. The same starting point can correspond to different $\Delta\sigma_0 = \sigma_{y_0} - \sigma_{z_0}$: $|\Delta\sigma_0| < (2/\sqrt{3})\sqrt{\sigma_T^2 - \sigma_{m_0}^2}$, $|\sigma_{m_0}| < \sigma_T$, where σ_T is the static yield stress. In the calculations, it was assumed that $\sigma_s = 3\sigma_T$.

2. Elastic loading occurs along the straight line

$$\sigma_x = ((1-\nu)/\nu)(\sigma_m - \sigma_{m0}), \text{ for } \nu=(1/3)\ \sigma_x=2(\sigma_m-\sigma_{m0}). \qquad (3)$$

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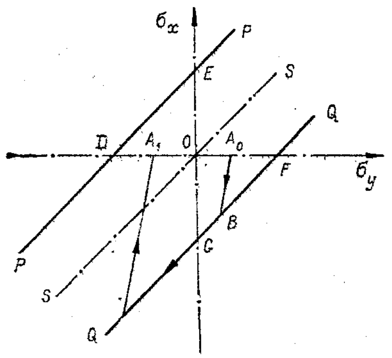

Fig. 1

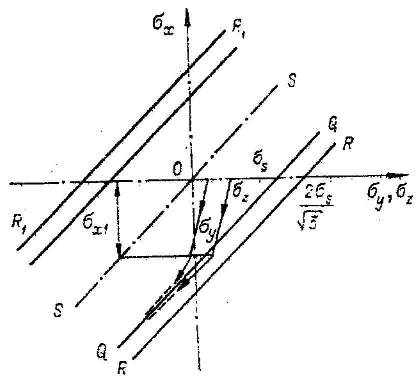

Fig. 2

3. The point at which the flow surface is reached is characterized by the quantity

$$\sigma_{x1} = \frac{1-\nu}{1-2\nu}\left(\frac{3}{2} S_{x1} + \sigma_{m0}\right), \tag{4}$$

where $S_{x_1} = -(1/3)\sqrt{4\sigma_s^2 - 3\Delta\sigma_0^2}$. For $\nu = 1/3$, $\sigma_{x_1} = 3S_{x_1} + 2\sigma_{m0}$.

4. Further loading is described by the following parametric equations, obtained by solving the system (2):

$$\frac{\Delta\sigma_x}{\sigma_s} = -\frac{2}{3}\left(\frac{\beta e^t - 1}{\beta e^t + 1} + t\right) - \frac{S_{x1}}{\sigma_s},$$
$$\frac{\Delta\sigma_m}{\sigma_s} = \frac{1}{3}\left(\frac{\beta e^t - 1}{\beta e^t + 1} - 2t\right) + \frac{S_{x1}}{2\sigma_s}, \tag{5}$$

where $\beta = (2\sigma_s - 3S_{x1})/(2\sigma_s + 3S_{x1})$; $t > 0$; $\Delta\sigma_x = \sigma_{xk} - \sigma_{x1}$; $\Delta\sigma_m = \sigma_{mk} - \sigma_{m1}$; σ_{xk} is the amplitude of the wave; σ_{mk} is the value of σ_m corresponding to σ_{xk}, while σ_{m1} corresponds to σ_{x1}.

The asymptotic dependence between σ_{xk} and σ_{mk} for large amplitudes σ_{xk} can be obtained from Eq. (5) as well as relations (3) and (4): $\sigma_{xk} \to \sigma_{mk} - \sigma_s$. For this reason, the loading trajectories in the plastic region in the coordinates $\sigma_x - \sigma_m$ have the form of the curve NR shown in Fig. 3.

Analysis of the loading according to the simplified scheme presumes that the trajectory KNR can be replaced by the broken trajectory KQG and the quantity $\Delta\sigma$ can be set equal to zero. The point P corresponds to an approximate unloading quantity σ'_{m2}; the point T corresponds to the exact σ_{m2}. We shall denote the correction PT in terms of σ_p, $\sigma_p > 0$.

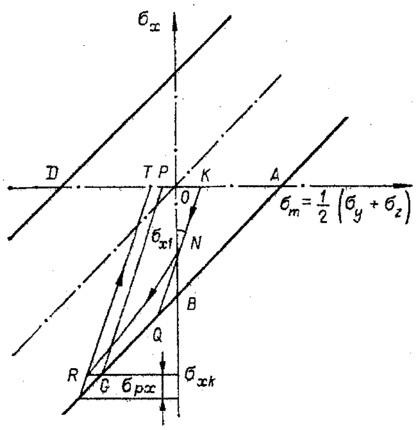

Fig. 3

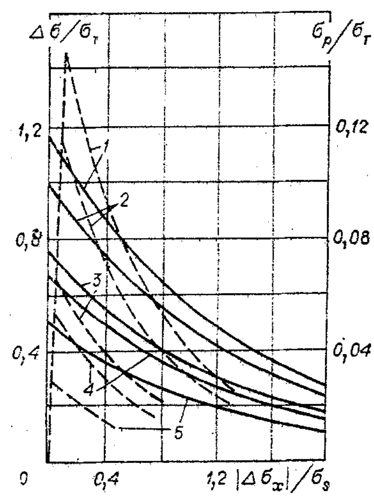

Fig. 4

TABLE 1

$\Delta\sigma_0$	0 (broken)	$\frac{1}{2}\sigma_T$	$\frac{2}{3}\sigma_T$	$\frac{3}{4}\sigma_T$	σ_T	$\frac{2}{\sqrt{3}}\sigma_T = 1{,}15\,\sigma_T$
$S_{x1} = -\frac{1}{3}\sqrt{4\sigma_s^2 - 3\Delta\sigma_0^2}$	$\frac{2}{3}\sigma_s$	$0{,}660\sigma_s$	$0{,}654\sigma_s$	$0{,}651\sigma_s$	$0{,}638\sigma_s$	$0{,}629\sigma_s$
$\beta = \frac{2\sigma_s - 3S_{x1}}{2\sigma_s + 3S_{x1}}$	∞	190	106	83,3	46,0	34,0
Line number in Fig. 4		5	4	3	2	1

To completely determine the stress state after unloading, it is necessary to know σ'_{m2}, σ_p, and the difference in stresses after unloading $\Delta\sigma_2 = \sigma_{y_2} - \sigma_{z2}$. The dependences of $\Delta\sigma$ and σ_p on $\Delta\sigma_x$ with $\nu = 1/3$, obtained by transforming system (5), have the following form in a parametric representation:

$$\frac{\Delta\sigma}{\sigma_T} = \pm 4\sqrt{3\beta}\,\frac{\exp(t/2)}{\beta e^t + 1},\quad \frac{\sigma_p}{\sigma_T} = \frac{6}{\beta e^t + 1},$$

where the sign in the first equation coincides with the sign of $\Delta\sigma_0$.

Figure 4 presents the computed corrections with the parameters indicated in Table 1. The dashed lines correspond to the quantity σ_p and the continuous lines to $\Delta\sigma$.

The algorithm for using the simplified scheme with the corrections is as follows. The quantities σ_{y0} and σ_{z0} are calculated from the initial stresses $\sigma_{m0} = (1/2)(\sigma_{y0} + \sigma_{z0})$ and $\Delta\sigma_0 = \sigma_{y0} - \sigma_{z0}$. Then, taking into account the known amplitude of the wave σ_{xk}, an approximate unloading value of σ'_{m2}, (point P) is found. Then the value of $\Delta\sigma_x = \sigma_{xk} - \sigma_{x_1}$ is determined. Using the graphs presented in Fig. 4 and $\Delta\sigma_x$, it is possible to obtain the values of σ_p and $\Delta\sigma_2$ and then σ_{y2} and σ_{z2}

$$\sigma_{y2} = \sigma'_{m2} + (1/2)\,\Delta\sigma_2 - \sigma_p,\quad \sigma_{z2} = \sigma'_{m2} - (1/2)\,\Delta\sigma_2 - \sigma_p.$$

It is evident in Fig. 4 that the most important correction is $\Delta\sigma$ and the quantity σ_p can be neglected for $\Delta\sigma_x < -\sigma_s$.

If $\sigma_{m2} < -\sigma_T$, then after the shock-wave process, the point T begins to drift comparatively slowly (see Fig. 3) toward increasing σ_m. A calculation performed based on the associated law of flow (1) and the conditions for uniaxial deformation $\varepsilon_y = \varepsilon_z = 0$, shows that the final state σ_{m3} and $\Delta\sigma_3$ is determined by the system of equations:

$$\sigma_{m3}^2 + (3/4)\,\Delta\sigma_3^2 = \sigma_T^2,\quad \Delta\sigma_3/\Delta\sigma_2 = (\sigma_{m3}/\sigma_{m2})^{\gamma},$$

where $\gamma = 3(1 - \nu)/(1 + \nu)$; for $\nu = 1/3$, $\gamma = 3/2$.

In the first approximation, we can assume that $\sigma_{m3} = -\sigma_T$ and $\Delta\sigma_3 = \Delta\sigma_2(-\sigma_T/\sigma_{m2})^{3/2}$.

In practical applications of the calculations presented above, it is important to know the correction σ_{px} (see Fig. 3). The latter represents the difference between the exact and approximate amplitude of the wave, necessary for obtaining a definite finite state σ_{m2} characterized by the point T.

The value of σ_{px} can be determined from the transcendental equation $\sigma_{px}/\sigma_s = 4/[1 + \beta \exp(\alpha - \sigma_{px}/\sigma_s)]$, where $\alpha = 2 + (3/\sigma_s)(S_{x1} + \sigma_{m0} - \sigma_{m2})$, $\sigma_{m2} < \sigma_{m0}$.

The calculation performed with $\sigma_{m2} = 0$ shows that for $\sigma_s = 3\sigma_T$, $\max(\sigma_{px}/\sigma_s) = 0.114$, which is less than 6% of the amplitude of the wave σ_{xk}, equal in this case to $2\sigma_s$. For this reason, in practical calculations, this correction may be neglected. This is a result of the fact that an insignificant relative change in wave amplitude σ_{xk} leads to larger changes in the final state, σ_{m_2}/σ_T, since usually $\sigma_{xk} = 6\sigma_T$.

We should compare the characteristics of the dynamic shock wave process, taken into account in this work, with the quasistatic loading and unloading conditions. As shown (see, for example, [5, 6]), the real high-velocity pattern of the deformation of a viscoelastoplastic substance can be successfully approximated by the usual elastoplastic quasistatic process, characterized by some effective dynamic yield stress, depending on the strain rate and the viscosity of the material.

For real strain rates, reached in the process of removing residual stresses by explosive working of such materials as, for example, St. 3 steel, the dynamic yield stress is 3-4 times greater than the static value. These data, based on experiments with high-speed stretching and compression of rods, are presented in [7].

In addition, the characteristic feature of the dynamic nature of deformation is included by introducing into the analysis the drift of the stressed state of the substance toward the static yield stress.

In conclusion, we thank Yu. I. Fadeenko for his assistance in this work and for discussion of the results.

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RELAXATION OF SUBMICROSECOND PRESSURE PULSES IN A SOLID

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Stress relaxation in dynamical problems of plasticity is described, from the standpoint of dislocation dynamics, by the Sokolovskii–Malvern–Duvall equation [1]:

$$\partial\sigma_{ij}/\partial t - \rho c^2 \partial\varepsilon_{ij}/\partial t = - a\partial\varepsilon^p_{ij}/\partial t \quad (1)$$

which takes into account the effect of velocity on the nature of the wave motion of the deformation. The plastic strain rate tensor $\dot{\varepsilon}^p_{ij}$ is written as the result of the simultaneous gliding in opposite directions of postive and negative dislocations

$$\dot{\varepsilon}^p_{ij} = \sum_{m=1}^{M} \left[{}^+\alpha^{(m)}_{ik} e_{jkl}\,{}^+v^{(m)}_l + {}^-\alpha^{(m)}_{ik} e_{jkl}\,{}^-v^{(m)}_l\right], \quad (2)$$

where the summation is over all the slip planes; ${}^+\alpha^{(m)}_{ik}$ and ${}^-\alpha^{(m)}_{ik}$ are the positive and negative dislocation density tensors.

As a rule, the conditions of deformation at strain rates $\dot{\varepsilon} < 10^3$ ensure, on the average, equality of the positive and negative dislocations, which corresponds to a zero net Burgers vector of the dislocation structure. In the case of pulse or shock loading, however, these conditions may not be fulfilled. In accordance with the definition of the dislocation density tensor in continuum dislocation theory, the latter is written in terms of plastic distortion gradients in the form $\alpha_{ij} = -e_{ikl}\nabla_k w_{lj}$. This means that in the presence of the large displacement gradients realized under high-speed loading the absolute values of the charge dislocation density may also be large. As shown in [2, 3], especially favorable conditions for the appearance of dislocation charges are realized in the contact loading zone.

As is known, charge dislocations are sources of long-range internal stress fields in

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